

ACOUSTOELASTIC RESPONSES OF AN ELASTOPLASTICALLY DEFORMED BODY*

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INTRODUCTION

The acoustoelastic response of an elastically deformed body can be characterized successfully by the theory of acoustoelasticity [1]. In the case of a body with plastic deformation, however, many investigators [2-5] have shown that the hyperelastic theory of acoustoelasticity is inadequate, and several modified theories of acoustoelasticity [6-10] were then proposed to incorporate the plastic strain effect.

Uniaxial tests in the plastical range were conducted to investigate the plastic strain effect on the change of acoustobirefringence (difference of two shear wave speeds polarized along two perpendicular directions) on carbon steel specimens (C:0.18%). The theory proposed by Pao and Gamer [9] which is independent of any yield condition of plasticity is adopted in this paper. In the theory, the effects of texture changes caused by plastic strain is characterized by two texture constants in the theory of acoustoelasticity for an initially isotropic body. These texture constants of the carbon steel specimen are determined from the uniaxial test results.

THEORETICAL BACKGROUND

A modified theory of acoustoelasticity for an elastoplastically deformed body as proposed by Pao and Gamer [9,10] is based on the theory of small motion superimposed on a predeformed body. The equation of motion for the incremental deformation u in the initial coordinate X can be written as [9]

$$\frac{\partial}{\partial X_K} \left(T_{JK} + \frac{\partial u_J}{\partial X_L} \sigma_{KL}^i \right) = \rho^i \frac{\partial^2 u_J}{\partial t^2}, \quad (1)$$

where σ_{KL}^i is the initial stress or residual stress inside the predeformed

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body, ρ^i is the mass density in the initial state. The incremental stress, T_{JK} , is defined as the difference between the second Piola-Kirchhoff stress in final state T_{JK}^f and the Cauchy stress in the initial state σ_{KL}^i

$$T_{JK} = T_{JK}^f - \sigma_{JK}^i \quad (2)$$

The equation of motion (Eq. 1) is valid for a general form of predeformation, infinitesimal or finite, elastic or inelastic, for it is obtained without invoking any constitutive relation.

For an elastoplastically deformed body, the constitutive relation for the incremental infinitesimal strains e_{KL} was postulated by Pao and Gamer [9] as

$$T_{IJ} = \bar{C}_{IJKL} e_{KL} \quad (3)$$

where the total strain is decomposed into an elastic part e^e and a plastic part e^p , and

$$\begin{aligned} \bar{C}_{IJKL} = & C_{IJKL}^r (1 - e_{NN}^e) + C_{MJKL}^r \frac{\partial u_I^e}{\partial X_M} + C_{IMKL}^r \frac{\partial u_J^e}{\partial X_M} \\ & + C_{IJML}^r \frac{\partial u_K^e}{\partial X_M} + C_{IJKM}^r \frac{\partial u_L^e}{\partial X_M} + C_{IJKLMN}^r e_{MN}^e \end{aligned} \quad (4)$$

The C_{IJKL}^r and C_{IJKLMN}^r are the elastic moduli at the relaxed state (A state after removing all the residual stresses). For a body that is isotropic at the natural state they assume the following form,

$$\begin{aligned} C_{IJKL}^r = & \lambda \delta_{IJ} \delta_{KL} + \mu (\delta_{IK} \delta_{JL} + \delta_{IL} \delta_{JK}) + \lambda' (e_{IJ}^p \delta_{KL} + \delta_{IJ} e_{KL}^p) \\ & + \mu' (e_{IK}^p \delta_{JL} + \delta_{IK} e_{JL}^p + e_{IL}^p \delta_{JK} + \delta_{IL} e_{JK}^p) \end{aligned} \quad (5)$$

where λ' and μ' are two new material constants. The third order constants C_{IJKLMN}^r are approximated by those at the natural state,

$$C_{IJKLMN}^0$$

With the new constitutive equation (Eq. 3), the birefringent formula in a naturally isotropic body with biaxial plastic strains and stresses is

$$\frac{\Delta V}{V_0} = \frac{V_{T1} - V_{T2}}{V_0} = \frac{\mu'}{2\mu} (e_1^p - e_2^p) + \frac{1}{2\mu} \left(1 + \frac{\nu_3}{\mu}\right) (\sigma_1^1 - \sigma_2^1) \quad (6)$$

where μ , ν_3 are one of the second and third order constants respectively. V_{T1} , V_{T2} are the shear wave speeds polarized along the principal stress directions X_1 , X_2 respectively.

In the case of uniaxial testing, the principal stress σ_1^i and principal plastic strain e_1^p are measured along the longitudinal axis of the specimen, and $\sigma_2^i = 0$. The plastic strain e_2^p equals $-\frac{1}{2} e_1^p$, if the volumetric change of plastic deformation is assumed to be zero ($e_1^p + e_2^p + e_3^p = 0$). Thus, the birefringent formula (Eq. 6) reduces to

$$\frac{\Delta V}{V_0} = \frac{3}{2} C_t e_1^p + \frac{1}{2\mu} \left(1 + \frac{\nu_3}{\mu}\right) (\sigma_1^i - \sigma_2^i) , \quad (7)$$

where $C_t = \mu'/\mu$ is the acoustoplastic coefficient.

BIOT'S THEORY OF INCREMENTAL DEFORMATION

Recently, Man and Lu [11] critically examined various versions of the theory of acoustoelasticity and presented a new theory. Their theory is an extension of Hoger's theory of residual stress [12], which in turn is based on Biot's theory of incremental deformation proposed in 1940's. Biot's theory as presented in the monograph at a later date [13] consists of two parts, the equation of motion (or equilibrium) for a body with initial deformation (initial stresses and strains) and the constitutive equation for the incremental stresses. The former is essentially the same as Eq. (1), the latter, however, is somewhat different from Eqs. (3) and (4).

Biot developed his theory in terms of the Cauchy stress σ^i , and the first Piola-Kirchhoff stress P^f . In analogy to Eq. (2), the incremental first Piola-Kirchhoff stress is defined as

$$P = P^f - \sigma^i . \quad (8)$$

This incremental stress, however, is composed of two parts, the one due to the rotation of initial stress σ^i , and the other one due to the pure deformation τ . If the antisymmetric part of the displacement gradient is represented by w , which corresponds to rotation in the infinitesimal deformation, we have the following relation

$$P = w\sigma^i + \frac{1}{2}(\alpha\sigma^i - \sigma^i\alpha) + \tau . \quad (9)$$

A constitutive relation can then be postulated for the pure deformation part of the incremental first Piola-Kirchhoff stress τ . This part is called the incremental elasticity tensor by Hoger [12] who reformulated Biot's theory in a concise form by applying the principle of material objectivity to the constitutive equation.

Hoger's theory was revised further by Man and Lu [11] by introducing another incremental elasticity tensor. The new incremental elasticity tensor was shown [14] to be equivalent to the incremental second Piola-Kirchhoff stress (Eq. 2). The formulations of Man and Lu's [11] and Pao and Gamer's theory [9,10] are equivalent, except that the latter consider further the plastic strain effect on the change of acoustoelastic response.

EXPERIMENTAL INVESTIGATION

To investigate the plastic strain effect on the change of acousto-birefringence, uniaxial tensile and compressive tests in the plastic range are performed. Carbon steel (C1018, C:0.18%) was chosen as the testing material. The specimens were cut from a large steel plate such that the loading direction of the specimen is parallel to the rolling direction of the plate. Four strain gages were mounted to monitor the uniaxial strain in the specimen. A home-made dual polarization shear wave transducer was used to measure the change of the birefringence during the loading process.

Shown in Figure 1 is the relation between the birefringence and the total compressive strain. The open circles represent the birefringences measuring during the loading process, and the solid circles represent those measured at the relaxed states where the external load has been totally removed. It is seen that the birefringences at the relaxed states are different from the inherent birefringence at the natural state (zero strain and zero stress).

Figure 2 shows the dependence of the change of birefringence on the uniaxial plastic strain (tensile and compressive). For small plastic strain (less than 0.5%), the dependency is approximated linear. At the relaxed states (solid circles), the uniaxial stresses are vanishing ($\sigma_1 = 0$), hence, the acoustoplastic coefficient of carbon steel is obtained by linear least square fitting as $C_t = 0.11$.

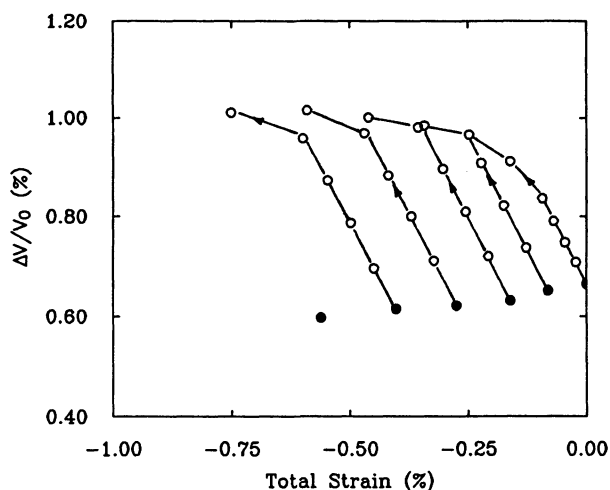


Fig. 1 Acoustoelastic birefringence history of C1018 steel specimen during compressive elastoplastic cyclic loading.

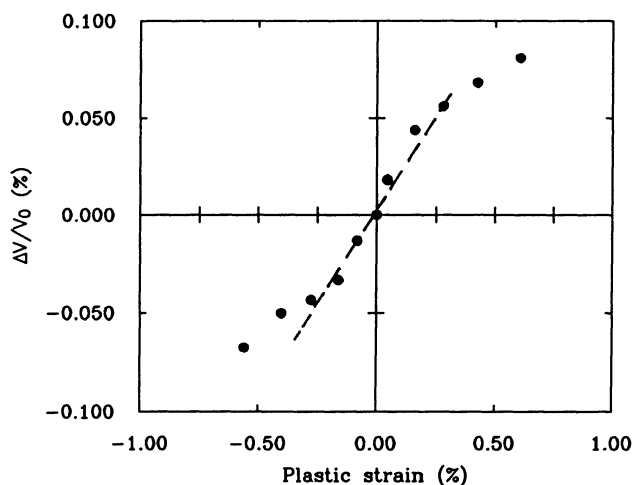


Fig. 2 The dependency of acoustobirefringence on plastic strain (tensile and compressive).

CONCLUSIONS

By postulating a constitutive relation for the incremental second Piola-Kirchhoff stress and infinitesimal strain, the texture effect of a plastically deformed body is incorporated into the theory of acoustoelasticity for small plastic strains. The slight change of the texture caused by plastic strain is characterized by the new material constants μ' , λ' . The acoustoplastic coefficient $C_t = \mu'/\mu$ is determined by conducting uniaxial tests in the plastically range. For a material with known second and third order constants and the acoustoplastic coefficient at the natural state, the acoustoelastic response of an elastoplastically deformed body can be predicted by Eq. 7.

Additional experimental results of acoustoplastic coefficients based on uniaxial and bending tests of steel specimen are reported in Ref. [14].

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